MATHEMATICS METHODS UNIT 1

Section Two:
Calculator-assumed

SOLUTIONS

Student Name: _____

Teacher Name: _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
                              correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
                              and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

(a) The points A and B have coordinates $(4, -6)$ and $(5, 8)$ respectively. If B is the midpoint of A and C, determine the coordinates of C. (3 marks)

Solution
If $C(x, y)$ then
$$\frac{4+x}{2} = 5 \Rightarrow x = 6 \text{ and } \frac{-6+y}{2} = 8 \Rightarrow y = 22$$
$$C(6, 22)$$
Specific behaviours
✓ writes equations for midpoints
✓ x-coordinate of C
✓ y-coordinate of C

(b) The points D and E have coordinates $(5p, -q)$ and $(2q, 3p)$ respectively, where p and q are constants. Determine the value of p and the value of q if the midpoint of D and E is at $(21, 17)$. (3 marks)

Solution
$$\frac{5p + 2q}{2} = 21 \text{ and } \frac{-q + 3p}{2} = 17$$
Solve simultaneously CAS to get $p = 10, q = -4$
Specific behaviours
✓ equations for both coordinates of midpoint
✓ value of p
✓ value of q

Question 10 **(8 marks)**

(a) The variables C and x are directly proportional and when $x = 5, C = 60$.

(i) Determine an equation for the relationship between C and x. (2 marks)

Solution
$C = mx, \qquad m = \dfrac{60}{5} = 12 \Rightarrow C = 12x$
Specific behaviours
✓ indicates use of line through origin
✓ correct relationship

(ii) State the value of C when $x = 15$. (1 mark)

Solution
$C = 12(15) = 180$
Specific behaviours
✓ correct value

(b) The time, t minutes, that a car takes to travel one kilometre at a constant speed of s kmh^{-1} is given by the formula $t = \dfrac{k}{s}$.
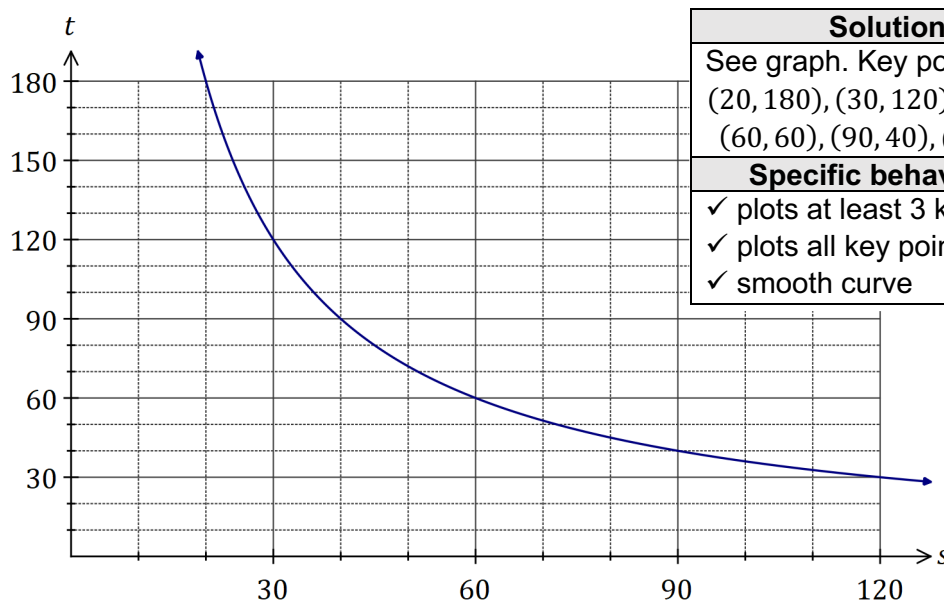
(i) Determine the value of the constant k, given that when $s = 40, t = 90$. (1 mark)

Solution
$90 = \dfrac{k}{40} \Rightarrow k = 3600$
Specific behaviours
✓ correct value

(ii) Determine the value of t when $s = 30$. (1 mark)

Solution
$t = 3600 \div 30 = 120$
Specific behaviours
✓ correct value

(iii) On the axes below, draw a graph to show how s varies with t. (3 marks)

Solution
See graph. Key points:
$(20, 180), (30, 120), (40, 90),$
$(60, 60), (90, 40), (120, 30)$
Specific behaviours
✓ plots at least 3 key points
✓ plots all key points
✓ smooth curve

Question 11 **(8 marks)**

The distortion of a signal, S, can be modelled by $S(x) = 4.55 - 4.5x + 1.95x^2 - 0.2x^3$, where x is the distance from the signal source in metres and $0 \leq x \leq 7$.
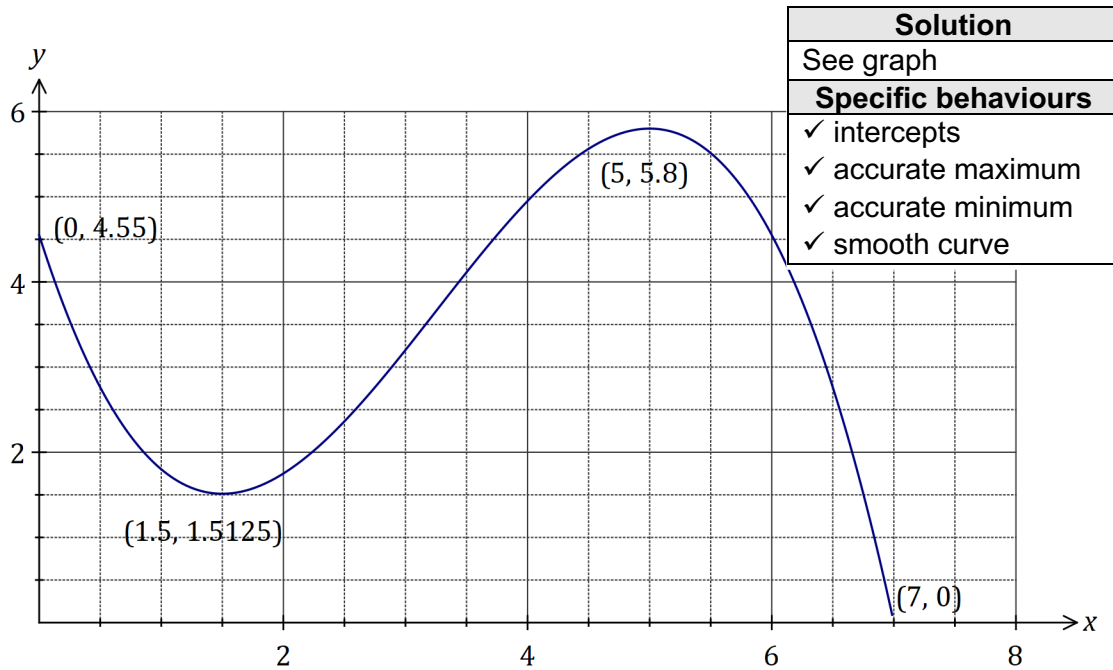
(a) Determine S when $x = 1$. (1 mark)

Solution
$S(1) = 1.8$
Specific behaviours
✓ correct value

(b) Draw the graph of $y = S(x)$ on the axes below. (4 marks)

Solution
See graph
Specific behaviours
✓ intercepts
✓ accurate maximum
✓ accurate minimum
✓ smooth curve

(c) Determine the equation of the straight line L that passes through the x-intercept and the y-intercept of the graph of $y = S(x)$. (2 marks)

Solution
$(0, 4.55)$ & $(7, 0) \Rightarrow m = -4.55 \div 7 = -0.65$
$y = -0.65x + 4.55$
Specific behaviours
✓ gradient
✓ y-intercept and equation

(d) Determine the coordinates of the point of intersection of L with the graph of $y = S(x)$ where $x > 0$ and $y > 0$. (1 mark)

Solution
$(2.75, 2.7625)$
Specific behaviours
✓ correct coordinates

See next page

Question 12 **(8 marks)**

(a) A parabola with equation $y = a(x - b)(x + c)$ has turning point at $(2, -8)$ and y-intercept at $(0, -6)$. The constants a, b and c are all positive.

(i) Determine the values of the positive constants a, b and c. (3 marks)

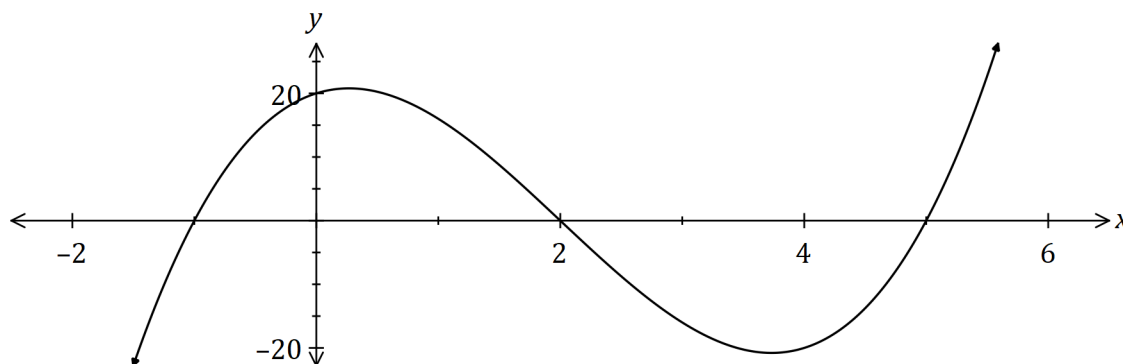| Solution |
|---|
| $y = a(x - 2)^2 - 8$ |
| $-6 = a(-2)^2 - 8 \Rightarrow a = 0.5$ |
| |
| $y = 0.5(x - 2)^2 - 8 = 0.5(x + 2)(x - 6)$ |
| |
| $a = 0.5, \quad b = 6, \quad c = 2$ |
| **Specific behaviours** |
| ✓ uses TP form to deduce a |
| ✓ uses CAS to obtain factored form |
| ✓ three correct values |

(ii) The parabola is translated 10 units to the left and 5 units downwards. Determine the equation of the transformed parabola in the form $y = a(x - p)^2 + q$.

(2 marks)

Solution
$y = 0.5(x - 2 + 10)^2 - 8 - 5$
$= 0.5(x + 8)^2 - 13$
Specific behaviours
✓ writes with correct value of p
✓ writes with correct value of q

(b) The graph of the cubic function $y = f(x)$ is shown below. Determine $f(10)$. (3 marks)

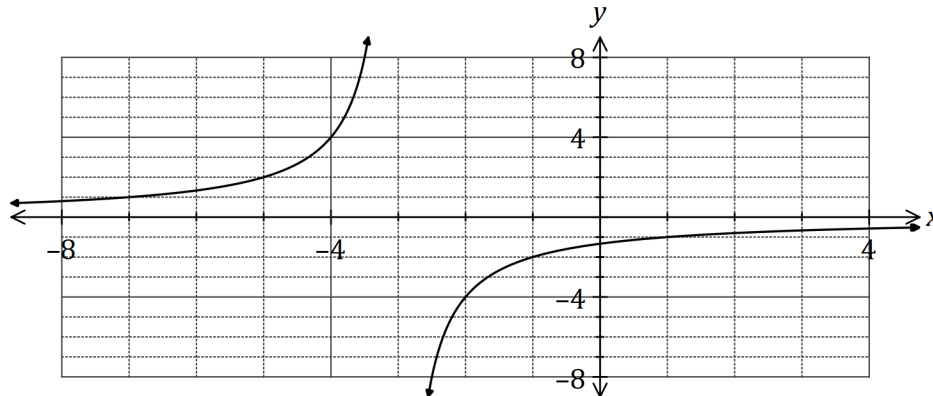| Solution |
|---|
| $f(x) = a(x + 1)(x - 2)(x - 5)$ |
| |
| $20 = a(1)(-2)(-5) \Rightarrow a = 2$ |
| |
| $f(10) = 2(11)(8)(5) = 880$ |
| **Specific behaviours** |
| ✓ cubic in factored form |
| ✓ correct value of a |
| ✓ required value |

See next page

Question 13 **(8 marks)**

The graph of $y = f(x)$ is shown below where $f(x) = \dfrac{a}{b - x}$.

(a) State the value of the constant a and the value of the constant b. (3 marks)

Solution
$b = -3$
Using $(-4, 4) \Rightarrow 4 = a \div (-3 + 4) \Rightarrow a = 4$
Specific behaviours
✓ value of b
✓ uses point on curve
✓ value of a

(b) The hyperbola shown above has two asymptotes. State their equations. (2 marks)

Solution
$x = -3, \qquad y = 0$
Specific behaviours
✓ vertical asymptote
✓ horizontal asymptote

(c) Describe how to transform the graph of $y = f(x)$ to obtain the graph of $y = f(x) + 1$ and state the domain and range of the transformed function. (3 marks)

Solution
Translate the graph 1 unit upwards.
Domain: $x \neq -3$
Range: $y \neq 1$
Specific behaviours
✓ transformation
✓ domain
✓ range

See next page

Question 14 **(8 marks)**

Not Assessed

Question 15 **(7 marks)**

An **obtuse** angled triangle WXY has $w = 45$ cm, $y = 34$ cm and an area of 739 cm^2.

(a) Sketch a triangle to show this information. (1 mark)

Solution
Specific behaviours
✓ triangle with all information

(b) Determine the size of $\angle X$. (2 marks)

Solution
$739 = \frac{1}{2}(34)(45)\sin X$
$X = 104.98 \approx 105°$
Specific behaviours
✓ substitutes into area equation
✓ correct (obtuse) angle

(c) Show that $x \approx 63$ cm. (2 marks)

Solution
$x^2 = 34^2 + 45^2 - 2(34)(45)\cos 104.98$
$x = 63.02 \approx 63$ cm
Specific behaviours
✓ uses appropriate equation that includes x
✓ substitutes correctly and solves to at least 1 dp

(d) Show that $\angle Y \approx 31°$. (2 marks)

Solution
$\dfrac{34}{\sin Y} = \dfrac{63.02}{\sin 104.98}$
$\angle Y = 31.4 \approx 31°$
Specific behaviours
✓ uses appropriate equation that includes Y
✓ substitutes correctly and solves to at least 1 dp

Question 16 **(8 marks)**

Not Assessed

Question 17 **(7 marks)**

Not Assessed

Question 18   **(8 marks)**

(a)   The equation of the axis of symmetry for the graph of $y = 3x^2 + 6x + 7$ is $x = k$.
Determine the value of k, using a method that does not refer to the graph of the parabola.

(2 marks)

Solution	
$x = -\dfrac{6}{2 \times 3} = -1$ $k = -1$	$y = 3(x + 1)^2 + c$ $\therefore k = -1$
Specific behaviours	
✓✓ value of k	

(b)   A parabola with equation $y = ax^2 + bx + c$ has a turning point at $(6, -5)$ and passes
through the point $(-2, -37)$. Determine the value of a, the value of b and the value of c.

(3 marks)

Solution
$y = a(x - 6)^2 - 5$ $-37 = a(-2 - 6)^2 - 5 \Rightarrow a = -0.5$ $y = -0.5(x - 6)^2 - 5$ $\quad = -0.5x^2 + 6x - 23$ $a = -0.5, \qquad b = 6, \qquad c = -23$
Specific behaviours
✓ solves for a using point in turning point form   ✓ expands and states b value correctly   ✓ expands and states c value correctly

(c)   Determine the value of the discriminant for the quadratic equation $16x^2 - 24x + 9 = 0$ and
use it to explain how many solutions the equation $(x + 1)(16x^2 - 24x + 9) = 0$ will have.

(3 marks)

Solution
$d = (-24)^2 - 4(16)(9) = 0$ When $d = 0$, quadratic will have one solution.   Hence equation will have two solutions - one from linear factor and one from quadratic factor.
Specific behaviours
✓ value of discriminant   ✓ uses discriminant to say quadratic will have one solution   ✓ explains why equation has two solutions

Question 19 **(6 marks)**

(a) A shelf held a collection of 22 different books, of which 5 were encyclopedias, 10 were science fiction and the rest were poetry.

A random selection of 4 books is to be made from the shelf.

Determine the number of ways:

(i) this can be done. (1 mark)

Solution
$\binom{22}{4} = 7\,315$
Specific behaviours
✓ correct number

(ii) a selection can be made that will not contain any encyclopedias. (2 marks)

Solution
$\binom{17}{4} \times \binom{5}{0} = 2\,380$
Specific behaviours
✓ correct number to choose from
✓ correct number

(b) When $(x + by)^n$ is expanded, the fourth term is $80x^2y^3$. Determine:

(i) the value of n (1 mark)

Solution
$n = 5$
Specific behaviours
✓ correct number

(ii) the value of b. Show your working. (2 marks)

Solution
$\binom{5}{2} b^3 = 80$
$b = 2$
Specific behaviours
✓ correct equation
✓ correct number

Question 20 **(8 marks)**

Not Assessed

Question 21 **(8 marks)**

Not Assessed

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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